properties of parallel lines cut by a transversal, perpendicular and angle bisectors, triangles, special quadrilaterals and circles

Parallel Lines

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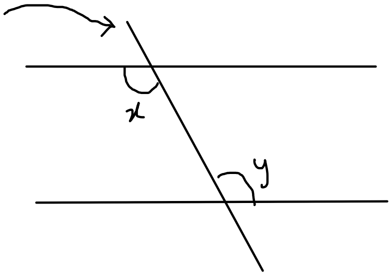
transversal

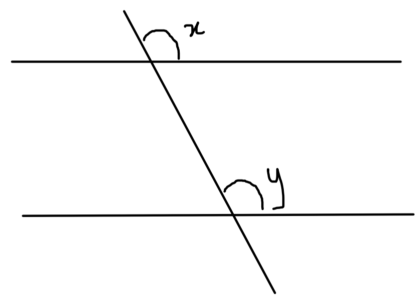


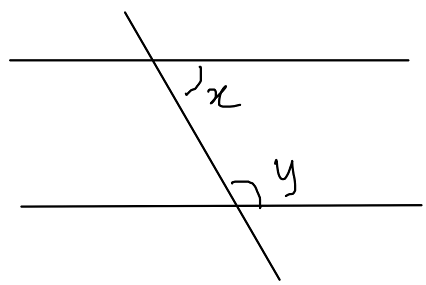
Alternate angles

$x = y$

Corresponding angles

$x = y$



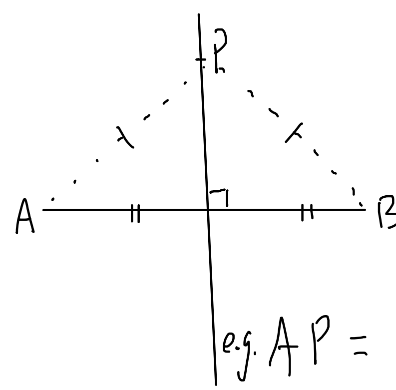


Interior Angles

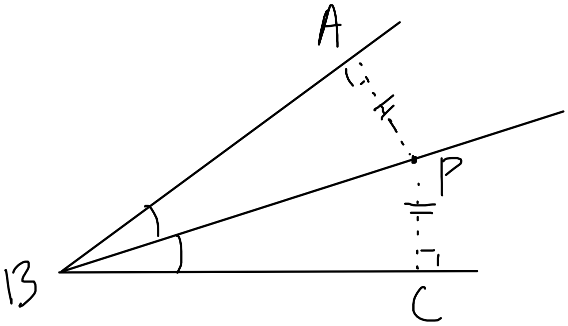
$x + y = 0$

Perpendicular bisector

- divides a line segment into half (e.g. AB)
- is perpendicular to the segment
- any point on it is equidistant from both ends of the segment



e.g. $AP = BP$



e.g. $AP = PC$

Angle bisector

- divides an angle into half (e.g. $\angle ABC$)
- any point on it is equidistant from the 2 lines of the angle

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Quadrilaterals in Circle

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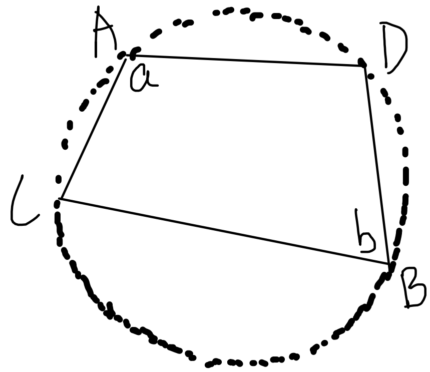
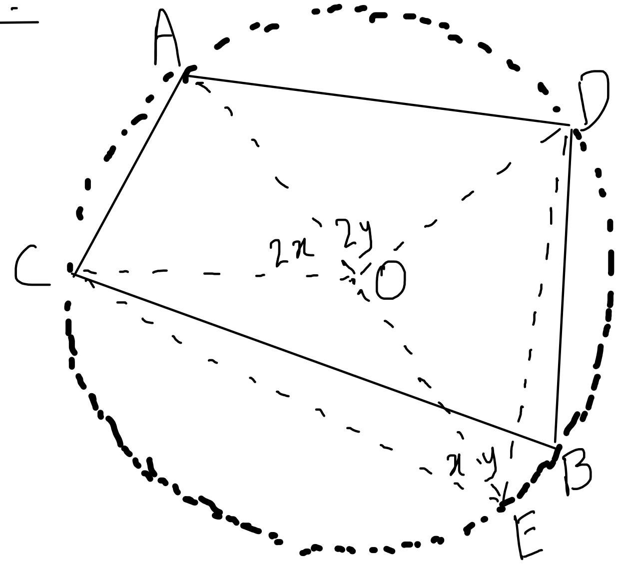


Sum of Opposite angles in quadrilateral in a circle is 180°.

$a + b = 180^\circ$

Proof:



Let O be centre, and AOE a straight line.

Angle at centre $= 2\times$ angle at circum.

$\Rightarrow \angle AOC = 2x$

$\angle AOD = 2y$.

$\triangle OAC$ isosceles $\Rightarrow \angle OAC = 90^\circ - x$

$\triangle OAD$ " $\Rightarrow \angle OAD = 90^\circ - y$

$\Big\}\ \angle CAD = 180^\circ - x - y$

Angles on same segment equal $\Rightarrow \angle CDB = x + y$

$\angle CAD + \angle CBD = 180^\circ - x - y + x + y$

$\therefore \quad a + b = 180^\circ$ //

Congruent Triangles

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SSS side side side

SAS side angle side

ASA angle side angle

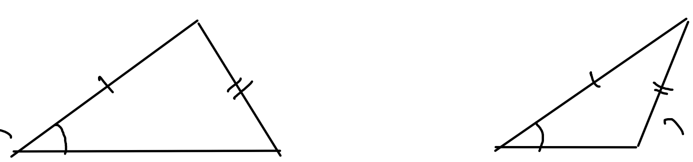
AAS angle angle side

AAA X

can be bigger or smaller

SSA X

non-included angle



also can.

Similar Triangles

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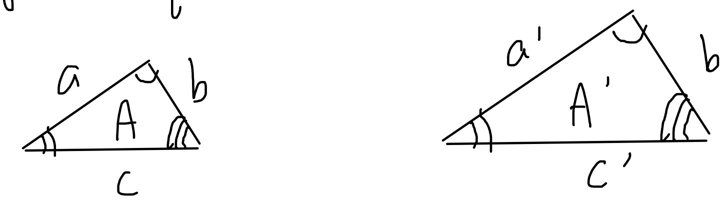
All angles equal



Corresponding sides' ratios equal: $\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$.

Areas' ratio equals square of ";"

$$\frac{A'}{A} = \left(\frac{a'}{a}\right)^2$$

Midpoint Theorem

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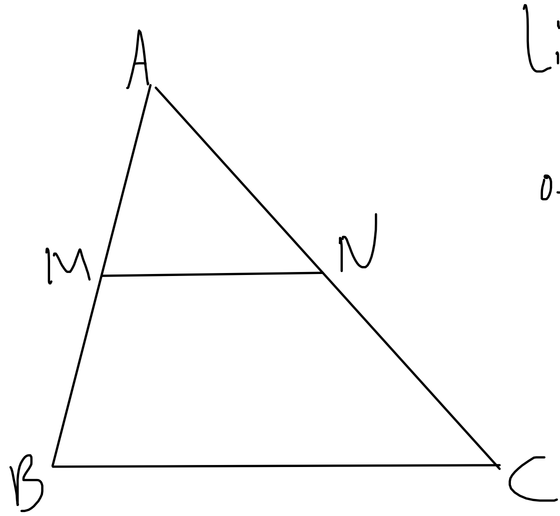

Line joining midpoints (e.g. M, N)
of 2 sides of a Δ
is half of the remaining side.

$$MN = \frac{1}{2} BC.$$

tangent-chord theorem (alternate segment theorem)

Alternate Segment Theorem

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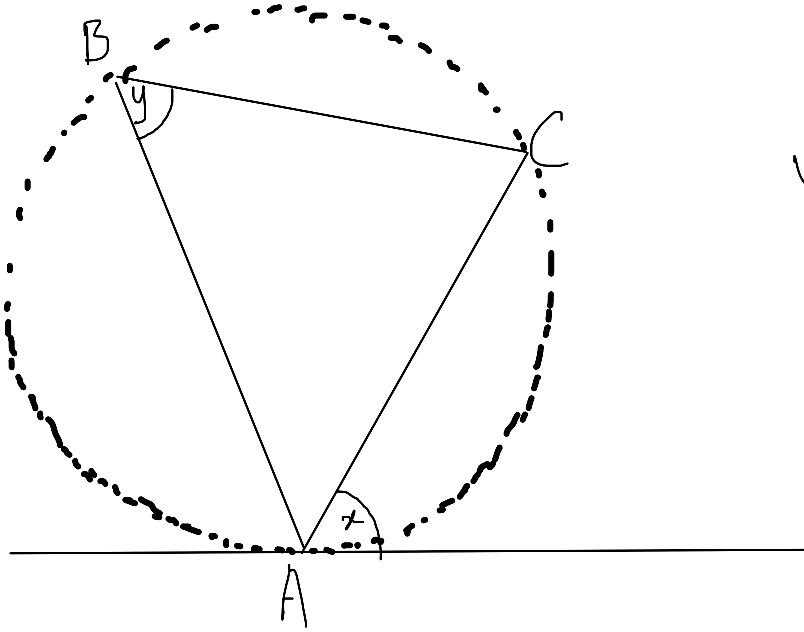
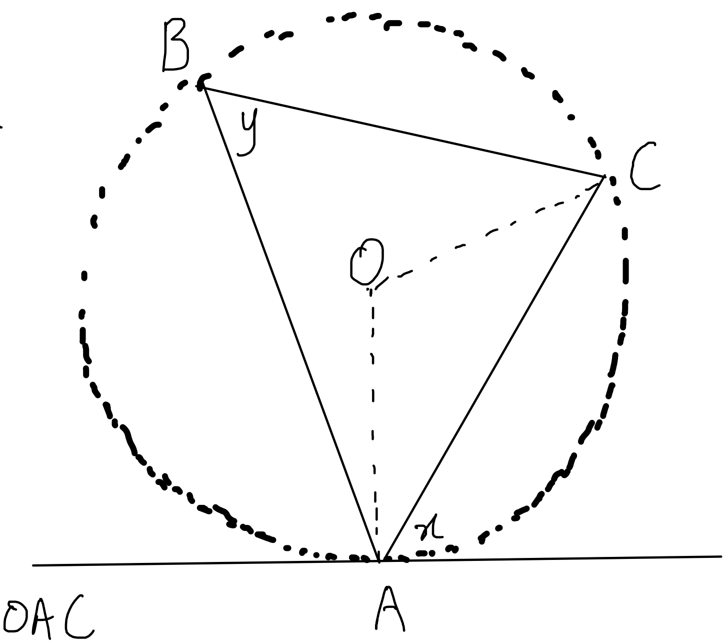


$y = x$.



Proof

Let O be centre.

$OA \perp$ tangent at A.

$\Rightarrow \angle OAC = 90° - x$

$\triangle OAC$ isosceles.

$$\Rightarrow \angle AOC = 180° - 2\angle OAC$$
$$= 2x$$

Angle at centre = 2× Angle at Circumference

$$\Rightarrow \angle AOC = 2y$$
$$2x = 2y$$
$$\therefore y = x$$

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Problem

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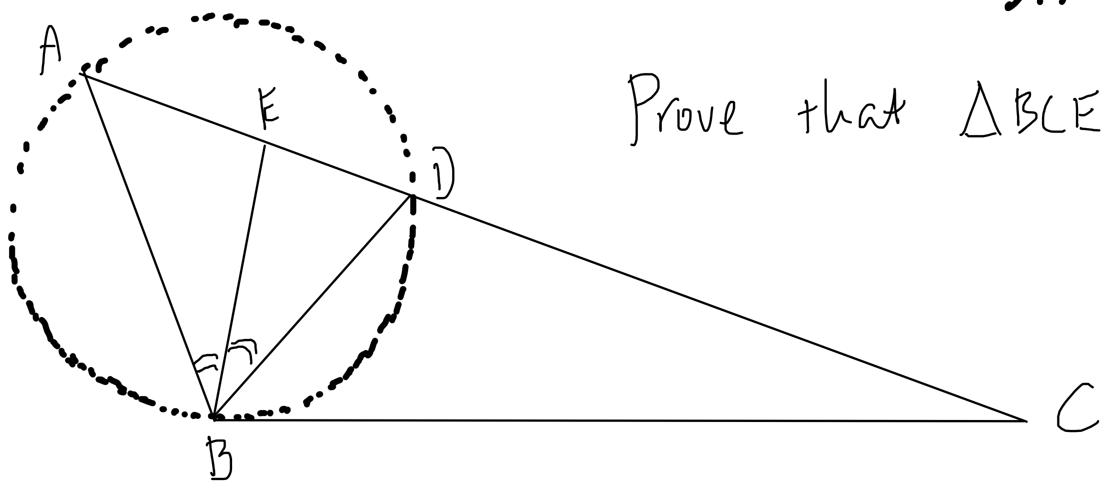
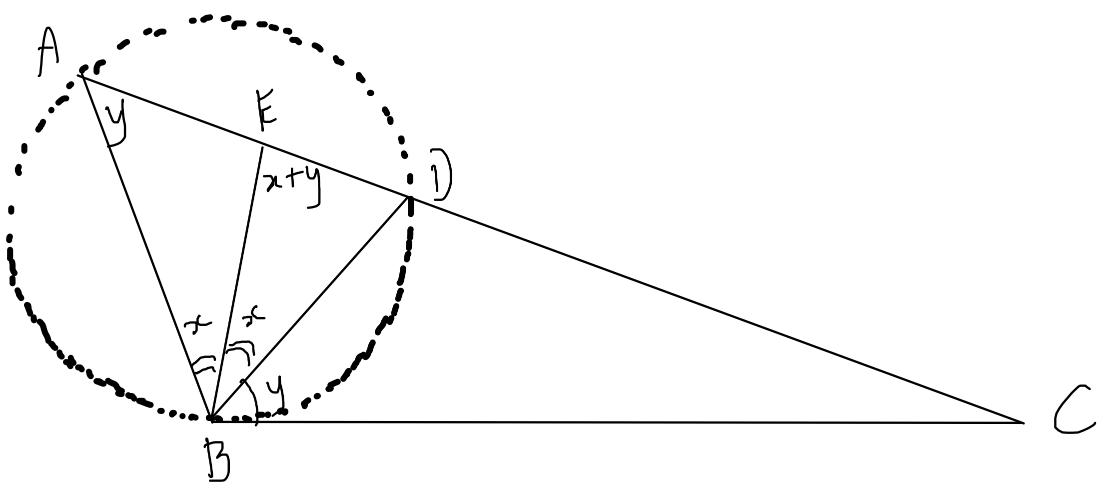
2013 P2 Q2



Prove that $\triangle BCE$ is isosceles.

Solution.



Let $\angle ABE = x$. Given $\angle EBD = x$.

Let $\angle DBC = y$.

Alternate segment $\Rightarrow$ $\angle BAD = y$

$\angle BED = x + y$ $\because$ exterior $\measuredangle$ of $\triangle$ = sum of interior opposite angles

So $\angle BEC = \angle CBE$

$\therefore$ $\triangle BCE$ is isosceles.